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PRODUCTION OF MESON PAIRS INVOLVING $L \neq 0$ MESONS IN PHOTON-PHOTON COLLISIONS *

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We present a formalism for studying the exclusive production or decay of mesons with any value of the internal orbital angular momentum L . As an application, we discuss the production of meson pairs (involving tensor and pseudotensor mesons) in photon-photon collisions.

1 Introduction

In a recent paper¹ we have presented a theoretical approach for the calculation of exclusive processes involving the production or decay of $(q\bar{q})$ mesons with any orbital angular momentum L . Starting from a bound-state model of weakly bound quarks, a formalism was derived that basically appears as a natural generalization of the usual perturbative QCD models for exclusive processes involving hadrons with zero orbital angular momentum.² As an interesting application, that formalism was used to study the production of meson pairs (involving $L \neq 0$ mesons) in photon-photon collisions. Predictions were given for the corresponding integrated cross sections in kinematical situations like those of LEP2 or future B-factories.

Here we will give a shortened presentation of this analysis, skipping most of the technical details of the approach and focusing on the qualitative features

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and on the key steps of the calculations. The interested reader can find a more detailed treatment in Ref. 1.

2 A generalization of PQCD models for exclusive processes to the production of $L \neq 0$ ($q\bar{q}$) mesons.

Before presenting our generalized approach, let us briefly recall the basic ideas underlying perturbative QCD models for exclusive processes (See Ref. 2 for more details). These models rely on factorization ideas, applied at the level of the helicity amplitudes for the physical process considered. Those amplitudes are then expressed as a convolution among: *i*) a hard-scattering amplitude involving the valence partons of all participating hadrons, assumed to be, for each hadron, collinear among themselves and with the parent hadron; *ii*) soft, non-perturbative contributions, described by means of (distribution) amplitudes for: *a*) finding the (collinear) valence partons in the incoming hadrons; *b*) the final partonic state to form the observed outgoing hadrons.

Notice that: *i*) only leading Fock states are considered (i.e., $|q\bar{q}\rangle$ for mesons, $|qqq\rangle$ for baryons); *ii*) valence partons are taken as massless and in a relative collinear configuration ($L = 0$). A number of higher-twist effects can in principle modify the details of the models and play a relevant role at presently accessible energies.

Let us now discuss our generalized approach; consider a production process $a(\lambda_a)b(\lambda_b) \rightarrow Q(LSJ\Lambda)c(\lambda_c)$, where Q is a ($q\bar{q}$) meson (with quantum numbers L, S, J , and Λ) and a, b, c are any particles (with helicities resp. $\lambda_a, \lambda_b, \lambda_c$). Starting from a bound-state model of weakly bound quarks for Q , a prescription³ relates the (hadronic-level) amplitude $\mathcal{M}_{\lambda_a\lambda_b\lambda_c}(E, \Theta)$ to the (partonic-level) amplitude $\mathcal{T}_{\lambda_a\lambda_b\lambda_c}^{S\Lambda_S}(E, \Theta, \mathbf{k}, x)$:

$$\mathcal{M}_{\lambda_a\lambda_b\lambda_c}(E, \Theta) = \left(\frac{M_Q}{2}\right)^{1/2} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \Psi^*(\mathbf{k}) \frac{\mathcal{T}_{\lambda_a\lambda_b\lambda_c}^{S\Lambda_S}(E, \Theta, \mathbf{k}, x)}{[(m_q^2 + \mathbf{k}^2)(m_{\bar{q}}^2 + \mathbf{k}^2)]^{1/4}}, \quad (1)$$

where: E is the total energy in the c.m. frame of a, b ; Θ is the c.m. scattering angle; $M_Q, m_q, m_{\bar{q}}$ are resp. the masses of the Q meson, the quark q and the antiquark \bar{q} ; $2\mathbf{k}$ is the relative three-momentum of q, \bar{q} inside the meson Q , in the meson rest-frame; $\Psi(\mathbf{k})$ is the corresponding meson wavefunction in momentum space. Notice that in Eq. (1) the q, \bar{q} spinors are already combined to form a total spin state $|S, \Lambda_S\rangle$. In the spirit of PQCD models, we can now naturally generalize Eq. (1), defining the q, \bar{q} 4-momenta as follows:

$$q^\mu = xQ^\mu + k^\mu \quad \bar{q}^\mu = (1-x)Q^\mu - k^\mu, \quad (2)$$

where Q^μ is the Q meson 4-momentum, and k^μ is the 4-dimensional generalization of \mathbf{k} . By introducing the meson distribution amplitude $\Phi_N(x)$ (normalized to unity), Eq. (1) generalizes to

$$\mathcal{M}_{\lambda_a \lambda_b \lambda_c}(E, \Theta) = \frac{1}{(2M_Q)^{1/2}} \int \frac{d^3 \mathbf{k}}{(2\pi)^{3/2}} \Psi^*(\mathbf{k}) \int_0^1 \frac{dx \Phi_N^*(x)}{\sqrt{x(1-x)}} \mathcal{T}_{\lambda_a \lambda_b \lambda_c}^{S \Lambda_S}(E, \Theta, \mathbf{k}, x). \quad (3)$$

Using the well-known decompositions: $\Psi(\mathbf{k}) = R_L(k) Y_{L\Lambda_L}(\theta, \phi)$; and $|J, \Lambda\rangle = \sum C_{\Lambda_L \Lambda_S \Lambda}^{L S J} |L, \Lambda_L\rangle |S, \Lambda_S\rangle$ (the C 's are the usual Clebsch-Gordan coefficients), we can now proceed with two crucial steps in our derivation:

i) Assuming that in the a, b c.m. frame the meson Q is extreme-relativistic, i.e. $\eta = (M_Q/E) \ll 1$, one can easily show that the partonic amplitudes \mathcal{T} become independent of the azimuthal angle ϕ ; as a consequence, it must be $\Lambda_L = 0$ and $\Lambda = \Lambda_S$.

ii) One can notice from Eq. (3) that the integrand with respect to the absolute value of the relative q, \bar{q} 3-momentum, $k = |\mathbf{k}|$, can be expanded in increasing powers of k , the leading term being proportional to k^L ; assuming that $R_L(k)$ is sharply peaked towards $k \rightarrow 0$, and keeping only the leading term in the power expansion, one gets from Eq. (3)

$$\begin{aligned} \mathcal{M}_{\lambda_a \lambda_b \lambda_c}^{LSJ\Lambda}(E, \Theta) &= f_L^* C_{0\Lambda\Lambda}^{LSJ} \lim_{\beta \rightarrow 0} \frac{1}{\beta^L} \int \frac{d(\cos \theta)}{2} d_{0,0}^L(\theta) \\ &\times \int \frac{\Phi_N^*(x) dx}{\sqrt{x(1-x)}} \mathcal{T}_{\lambda_a \lambda_b \lambda_c}^{S \Lambda}(E, \Theta, \beta, \theta, x), \end{aligned} \quad (4)$$

where we have introduced the dimensionless variable $\beta = 2k/M_Q$, and the normalization constant f_L , which is connected in the usual way to the value at the origin of the L -th derivative of the radial wave function $R_L(r)$.

This equation is the basic result of our formalism. It can easily be checked that, for $L = 0$, Eq. (4) leads exactly to the same expression as provided by the usual PQCD models.

3 An application: meson pair production in photon-photon collisions

We start this section by briefly recalling the physical interest in studying hadron production in photon-photon collisions.

3.1 Hadron production in $\gamma\gamma$ collisions

Photon-photon collisions represent a very useful tool for the study of hadron production. Basically, the more attractive feature is the simple, clean initial state, involving only QED interactions, which allows one to concentrate on the final, hadronic state. This way, in fact, some of the more clean tests for PQCD models were proposed.² It is well known that exclusive $\gamma\gamma \rightarrow$ hadron processes can be studied in e^+e^- colliders. Let us briefly recall some basic properties of $\gamma\gamma$ processes in this context.

First of all, compare the one-photon annihilation (OPA) contribution to the two-photon radiation (TPR) one. Due to the photon quantum numbers, OPA allows to investigate C-odd hadronic final states. OPA processes are of order α^2 but, due to the virtual photon propagator involved, $\sigma_{OPA}(e^-e^+ \rightarrow X) \sim 1/s$ (where s is the lab. energy squared). On the contrary, TPR processes make possible to study C-even final hadronic states (being in this sense complementary to the OPA contribution). Moreover, even though they are of order α^4 , one finds that $\sigma_{TPR}(e^-e^+ \rightarrow e^-e^+X) \sim \ln^2(s/m_e^2)$: as a consequence, the TPR contribution already dominates over the OPA one at beam energies of a few GeV.

Whereas for a given beam energy the e^-e^+ kinematics for the OPA process is fixed, the continuous spectra of the photon beams in the TPR process allows simultaneous measurements at different $\gamma\gamma$ invariant masses. Notice however that this comes to the expenses of collectable statistics at a given invariant mass; moreover, due to the typical bremsstrahlung spectrum ($\sim 1/E_\gamma$) of the radiated photons, most of the TPR processes have low invariant masses.

The photon propagators in the TPR process cause the bulk of photons to be radiated nearly on mass-shell, at small angles relative to the beam. This means that e^-e^+ colliders effectively provide two colliding beams of quasi-real photons with luminosities comparable to those of the collider itself.

3.2 The $\gamma\gamma \rightarrow M\bar{M}$ process

Eq. (4) can be easily generalized to the process where two ($q\bar{q}$) mesons are produced, $ab \rightarrow QQ'$ (herefrom symbols without and with the “prime” are pertinent to Q and Q' mesons respectively). One needs only to substitute

particle c in the original derivation with the meson Q' and repeat the same steps described in the previous section for the Q meson. We present the corresponding result directly in the case where the initial particles are two real photons:

$$\begin{aligned}
& \mathcal{M}_{\lambda_\gamma \lambda'_\gamma}^{LSJ\Lambda, L'S'J'\Lambda'}(E, \Theta) = f_L^* f_{L'}^* C_{0\Lambda\Lambda}^{LSJ} C_{0\Lambda'\Lambda'}^{L'S'J'} \\
& \times \lim_{\beta, \beta' \rightarrow 0} \frac{1}{\beta^L \beta'^{L'}} \int \frac{d(\cos \theta)}{2} d_{0,0}^L(\theta) \int \frac{d(\cos \theta')}{2} d_{0,0}^{L'}(\theta') \\
& \times \int \frac{\Phi_N^*(x) dx}{\sqrt{x(1-x)}} \int \frac{\Phi_N^*(x') dx'}{\sqrt{x'(1-x')}} \mathcal{T}_{\lambda_\gamma \lambda'_\gamma}^{S\Lambda, S'\Lambda'}(E, \Theta, \beta, \beta', \theta, \theta', x, x').
\end{aligned} \tag{5}$$

We can now apply our model to the case of the production, in photon-photon collisions, of meson pairs involving tensor and pseudotensor mesons. In order to get numerical results, we need three basic ingredients of Eq. (5): *i*) the hard scattering amplitudes $\mathcal{T}_{\lambda_\gamma \lambda'_\gamma}^{S\Lambda, S'\Lambda'}$; *ii*) the normalization constants f_L ; *iii*) the meson distribution amplitudes $\Phi_N(x)$. Let us briefly describe how these quantities are fixed in our approach.

3.3 Evaluation of partonic amplitudes

Since the partonic amplitudes are of course independent of L (the valence constituents of each meson are in a collinear configuration), we can actually divide our strategy in three steps: *i*) Take the results for the partonic amplitudes as from the usual PQCD models for $L = 0$ mesons; these amplitudes have been evaluated originally by Brodsky and Lepage,⁴ and are given as functions of (among other variables) x and x' , the fraction of the meson momentum carried by quark q (q') inside the meson Q (Q'). *ii*) Make the substitutions: $x \rightarrow \tilde{x} = x + (\beta/2) \cos \theta$, $x' \rightarrow \tilde{x}' = x' - (\beta'/2) \cos \theta'$; it is not difficult to convince oneself that this corresponds exactly to the generalized procedure exposed in the previous section, in the limit $M/E \ll 1$ (here M indicates the Q or Q' mass). *iii*) Finally, perform a series expansion of the resulting amplitudes in powers of β , β' , keeping only the physically relevant terms (i.e., those in $\beta^L \beta'^{L'}$).

Even so, the expressions of those amplitudes are quite involved and resort to numerical integration is required for performing the convolution integrals.

3.4 Normalized meson distribution amplitudes, $\Phi_N(x)$

In order to check the dependence of our results on the meson distribution amplitudes, we have considered two indicative choices: *i*) the so-called nonrelativistic DA, $\Phi_N(x) = \delta(x-1/2)$. It leads to very simple convolution integrals, and in this case we can perform analytical calculations even for the resulting differential cross sections. *ii*) A generalization of the so-called asymptotic distribution amplitude,² that is $\Phi_N(x) = N_L x^{L+1} (1-x)^{L+1}$, where N_L is a factor ensuring the required normalization to unity. In particular, for $L = 1$, $L = 2$ mesons we have respectively $\Phi_N(x) = 30x^2(1-x)^2$, $\Phi_N(x) = 140x^3(1-x)^3$. Since we have considered also the production of hybrid meson pairs (that is, pairs made of a pion plus a pseudotensor mesons), we have also to choose a DA for the pion; as an example, in the following we always use the well-known Chernyak-Zhitnitsky DA,⁵ $\Phi_{N,\pi}^{CZ}(x) = 30x(1-x)(2x-1)^2$.

3.5 Normalization constants f_L

In order to make complete numerical predictions for the processes of interest, we need finally to fix the values of the normalization constants f_L appearing in Eq. (5). For the pion, we take the experimental value of the leptonic decay constant, $f_\pi \cong 93$ MeV, and use the relation $|f_0| = f_\pi/(2\sqrt{3})$.

For tensor and pseudotensor mesons we evaluate, using the same theoretical approach, the corresponding two-photon decay widths, $\Gamma(Q \rightarrow \gamma\gamma)$. Making use of the available experimental data for the masses and the two-photon decay widths of the f_2 , a_2 , f_2' and π_2 mesons, one gets estimates of the absolute values of the corresponding $f_{1,2}$ constants. Both the case of nonrelativistic and generalized asymptotic DA's have been taken into account.

4 Results

Once we have evaluated the helicity amplitudes for the hadronic process, $\mathcal{M}_{\lambda_\gamma \lambda'_\gamma}^{LSJ\Lambda, L'S'J'\Lambda'}(E, \Theta)$, we can give predictions for physical observables, such as the differential cross section with respect to the scattering angle Θ

$$\frac{d\sigma^{\gamma\gamma \rightarrow QQ'}(E, \Theta)}{d(\cos \Theta)} = \frac{\xi}{128\pi E^2} \sum_{\lambda_\gamma \lambda'_\gamma, \Lambda \Lambda'} |\mathcal{M}_{\lambda_\gamma \lambda'_\gamma}^{LSJ\Lambda, L'S'J'\Lambda'}(E, \Theta)|^2, \quad (6)$$

where meson masses have been neglected in the phase-space factor, and $\xi=1/2$ if Q, Q' are identical particles, $\xi=1$ otherwise.

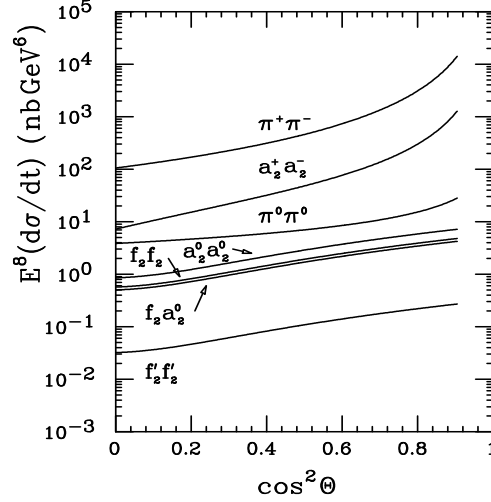


Fig. 1: Differential cross section $E^8 [d\sigma/dt]$ in $\text{nb} \times \text{GeV}^6$, as a function of $\cos^2 \Theta$, for the process $\gamma\gamma \rightarrow QQ'$ involving the production of tensor-meson pairs; the nonrelativistic DA was used for tensor mesons; for comparison, analogous curves for $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$, using the Chernyak-Zhitnitsky DA are also shown.

In Figs. 1-4 we are plotting the scaling differential cross sections $E^8 d\sigma/dt$, which is easily derived from Eq. (6) by noticing that, neglecting masses, $|t| = (E^2/2)(1 - \cos \Theta)$. It is also easy to derive from Eq. (6) the differential cross section with respect to the transverse momentum p_T of the outgoing mesons, $d\sigma^{\gamma\gamma \rightarrow QQ'}(E, p_T)/dp_T$. The integrated cross section for the overall process ($e^-e^+ \rightarrow e^-e^+QQ'$) can then be obtained, in the equivalent-photon approximation, by convoluting $d\sigma^{\gamma\gamma \rightarrow QQ'}/dp_T$ with the equivalent-photon spectrum of the two photons, and integrating over p_T from a given minimum value of p_T , p_T^{min} . In tables 1-3 we present the results for the integrated cross section for three cases: *i*) $\sqrt{s} = 200$ GeV (LEP2 energy), $p_T > 1$ GeV ; *ii*) $\sqrt{s} = 200$ GeV, $p_T > 2$ GeV ; *iii*) $\sqrt{s} = 10$ GeV (energy of a “B factory”), $p_T > 1$ GeV.

5 Conclusions

We have presented a formalism for the study of exclusive decay or production processes involving $(q\bar{q})$ mesons having non-zero orbital angular momentum. Our approach is a generalization of the usual perturbative QCD models for exclusive processes involving $L = 0$ mesons.

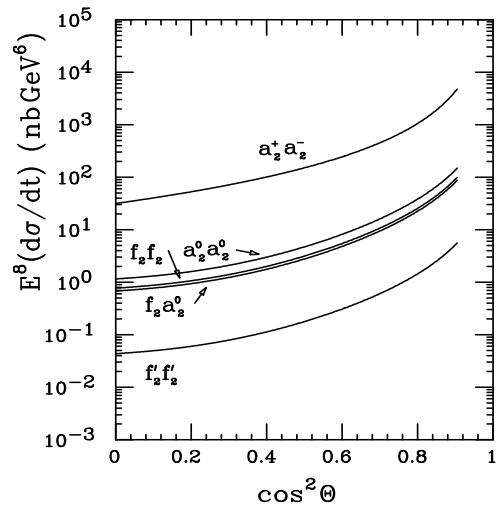


Fig. 2: The same as in Fig. 1, but the generalized asymptotic DA is used for tensor mesons.

As an application of our model, we have considered in detail the production of meson pairs (involving tensor, pseudotensor mesons) in photon-photon collisions. From fig.s 1-4 and tables 1-3 we can argue that the results obtained do not depend strongly on the distribution amplitude chosen for tensor and pseudotensor mesons: apart from $\pi_2^0 \pi_2^0$ production, the generalized asymptotic DA leads to approximately equal or slightly (at most by a factor of about 3) higher values, as compared to the nonrelativistic one. One can also notice that in general the charged channels give rise to significantly higher yields than the neutral ones, being thus more favorable for experimental searches.

Finally, tables 1-3 show that, although the integrated cross sections are small, there is some hope that the production of charged-meson pairs as here considered may become measurable with high-energy e^-e^+ colliders of the next generation, provided integrated luminosities as high as $\approx 10^{40} \text{ cm}^{-2}$ can be reached.

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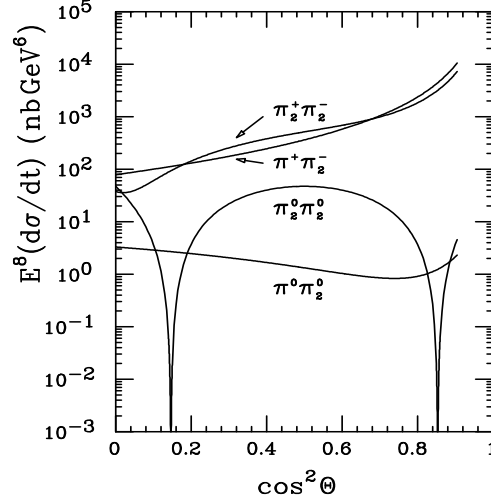


Fig. 3: Differential cross section $E^8 [d\sigma/dt]$ in $\text{nb} \times \text{GeV}^6$, as a function of $\cos^2 \Theta$, for the process $\gamma\gamma \rightarrow QQ'$ involving the production of pseudotensor-meson and hybrid (one pion plus one pseudotensor meson) pairs. The Chernyak-Zhitnitsky DA was used for pions, while the nonrelativistic DA was used for pseudotensor mesons.

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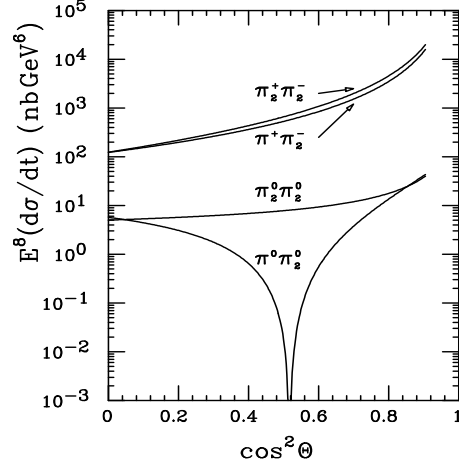


Fig. 4: The same as in Fig. 3, but the generalized asymptotic DA is used for pseudotensor mesons.

Table 1: Integrated cross sections (in 10^{-40} cm^2) of the process $ee' \rightarrow ee'QQ'$, for $\sqrt{s} = 200 \text{ GeV}$, $p_T > 1 \text{ GeV}$.

QQ'	$\sigma(ee' \rightarrow ee'QQ') [10^{-40} \text{ cm}^2]$	
	NR	GASY
$f_2 f_2$	35.2	49.1
$a_2^0 a_2^0$	49.7	92.6
$f_2 a_2^0$	31.1	57.5
$f_2' f_2'$	1.0	2.1
$a_2^+ a_2^-$	494.6	1721.8
$\pi_2^0 \pi_2^0$	236.1	37.7
$\pi_2^+ \pi_2^-$	1387.6	2698.2
$\pi^0 \pi_2^0$	165.8	389.6
$\pi^+ \pi_2^-$	6651.7	10476.8

Table 2: Same as table 1, but assuming $p_T > 2$ GeV.

QQ'	$\sigma(ee' \rightarrow ee'QQ') [10^{-40} \text{ cm}^2]$	
	NR	GASY
$f_2 f_2$	1.2	1.2
$a_2^0 a_2^0$	2.3	3.5
$f_2 a_2^0$	1.4	2.1
$f_2' f_2'$	0.1	0.1
$a_2^+ a_2^-$	20.3	68.9
$\pi_2^0 \pi_2^0$	21.6	4.1
$\pi_2^+ \pi_2^-$	92.6	164.2
$\pi^0 \pi_2^0$	6.5	13.4
$\pi^+ \pi_2^-$	189.6	298.4

Table 3: Same as table 1, but assuming: $\sqrt{s} = 10$ GeV, $p_T > 1$ GeV.

QQ'	$\sigma(ee' \rightarrow ee'QQ') [10^{-40} \text{ cm}^2]$	
	NR	GASY
$f_2 f_2$	2.3	2.6
$a_2^0 a_2^0$	3.6	5.9
$f_2 a_2^0$	2.3	3.7
$f_2' f_2'$	0.1	0.1
$a_2^+ a_2^-$	33.0	113.2
$\pi_2^0 \pi_2^0$	16.8	2.7
$\pi_2^+ \pi_2^-$	74.0	131.8
$\pi^0 \pi_2^0$	17.1	35.9
$\pi^+ \pi_2^-$	524.4	825.2